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Macroeconomics II

Lecture 06 March 2023

The Harrod-Domar model: assumptions and implications
Criticism by Solow

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Theoretical Lecture 6

Harrod-Domar model. The criticism by Solow

- **the hypotheses of the Harrod-Domar (H-D) model;**
- **the H-D model: definition, behaviour and reduced-form equation;**
- **criticisms of the hypotheses and unstable equilibrium of the H-D model;**
- **reactions and solutions by R. Solow.**

Reading:

Harrod, R.F. (1939), An Essay in Dynamic Theory, *The Economic Journal*, vol. 49, No 193, pp. 14-33

Louçã & Mortágua (2021), Chapter 12

Roy **Harrod** (1900 – 1978)

Evsey **Domar** (1914-1997)

Sir Roy F. Harrod
(1900-1978)

English economist, born in London. He studied at Oxford and, apart from a short break for service in World War II, and again as adviser to the International Monetary Fund in the early 1950s, he remained there, at Christ Church, throughout his career (1922-1967). He wrote the official biography of Keynes (1951), and wrote also on philosophy and logic as well as economics. He made several other discoveries more or less in parallel with others (the marginal revenue curve, the long-run envelope of short-run average cost curves, the theory of imperfect competition, the multiplier-accelerator model, and the IS-LM model). At last, another discovery brought him recognition: Harrod's "Essay in Dynamic Theory" (1939). The idea, which marked the beginning of the modern theory of growth, had also been developed by Domar, but at least Harrod got his name on the model this time. He was personal advisor to Winston Churchill during World War II.

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American economist. Born in Lodz, Russia (now Poland), he was raised and educated in Harbin, Manchuria, but moved permanently to the United States in 1936 and completed his studies there at the UCLA, Michigan, and Harvard, where he got his Ph.D. in 1947. He taught at several universities, including Johns Hopkins, before moving to MIT in 1958. He made contributions in at least three main areas of economics: economic growth, comparative economics, and economic history. His work on economic growth began with his 1944 model on government debt, which considered how economic growth can lighten the burden of the government debt. His major claim to fame, however, was in developing, parallel to Roy Harrod, a dynamic-equilibrium growth model (1946) as a way of extending the Keynesian demand-determined equilibrium into the long run.

Evsey D. Domar
(1914-1997)

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A Keynesian analysis in the models of economic growth

Roy **Harrod** (1939) and Evsey **Domar** (1946)

"Harrod and Domar expressed the dynamic relationship (*the effect of capital accumulation on growth*) in a simple equation, which neatly formalized, simplified and summarized the essence of almost 200 years' theorizing about economic growth" (Gylfason, 2003, p. 25)

Economic growth depends on three factors:

Savings rate of the households (S = s.Y)

Capital-output ratio (v = K/Y = ΔK/ΔY)

Depreciation rate (δ)

The assumptions of the Harrod-Domar (H-D) model

1. The technology

Production function with complementary factors (i.e., non-substitutable), with two (2) production factors: capital and labour

Technology with constant coefficients of the Leontief type (see previous lecture)

$Y = A.K = (1/v).K$, $v = K/Y$ (capital-output ratio) **constant**; $A = 1/v$ is capital productivity;

K (stock, €), Y (flow, €/year)

K/L constant (L not explicit in the model)
K and L grow at the same rate

Y/L constant (the growth rate of labour productivity is 0)
Y and L grow at the same rate

there is **no technological progress**

The assumptions of the Harrod-Domar (H-D) model (cont.)

2. Full employment
productive capacity of the economy is fully used; for a given K,

$Y = (1/v).K$

is the maximum GDP (potential output) that the economy may create, given the hypotheses and provided the technology is adequate (the required capital-output ratio) – otherwise, full employment is not reached

3. The economy is closed

4. Savings
total savings is a constant proportion of GDP
$S = s.Y$

5. Equality between savings and investment (a consequence of (3) above)
$I = S$

Harrod-Domar model: four equations

households/**consumers:**

(eq. 1) **$S = sY$,**

Total savings (S) equals the savings rate (s) times total income (Y)

firms/**producers:**

(eq. 2) **$K = vY$**

There is a fixed technical relationship between the capital stock and total output/income. That relationship is given by the capital-output ratio (v) or its reciprocal, the (marginal and average) productivity of capital (1/v).

definition of investment uses:

(eq. 3) **$I = \Delta K + \delta K \Leftrightarrow \Delta K = I - \delta K$**

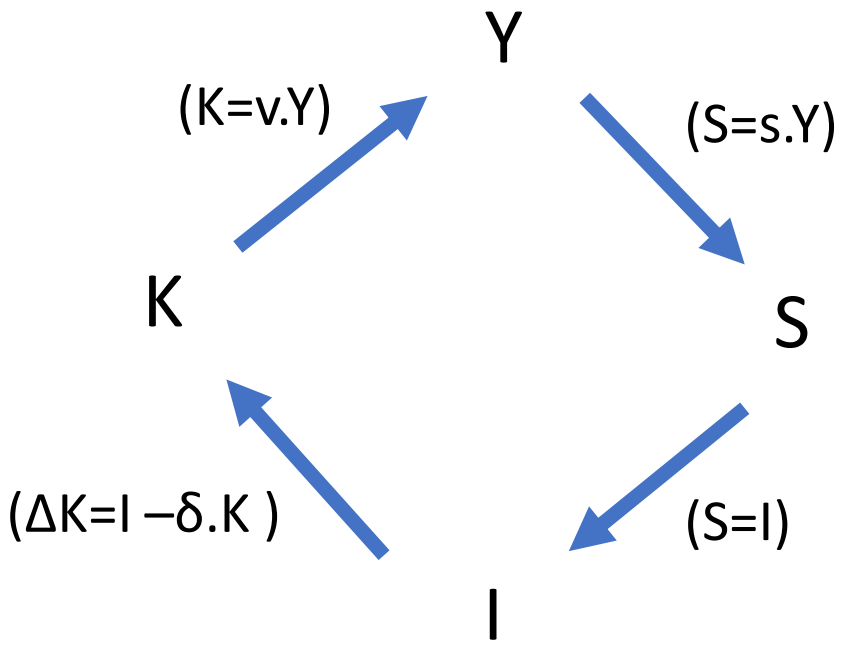
The change in the capital stock (ΔK) equals gross investment (I) minus the depreciation of the existing capital stock (δK).

Closed-economy **equilibrium** condition:

(eq. 4) **$S = I$**

Total savings equals total investment.

Harrod-Domar model: a circular logic ("accelerator effect")



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Harrod-Domar model: deducing the reduced-form equation

(Eq1) $S=s.Y$

(Eq2) $K=v.Y$

(Eq3) $I=\Delta K+\delta.K$ or $\Delta K=I-\delta.K$

(Eq4) $S=I$;

$\Delta K=I-\delta.K$ (eq.3)

$\Leftrightarrow \Delta K=S-\delta.K$ (eq.4)

$\Leftrightarrow \Delta K = s.Y-\delta.K$ (eq.1)

$\Leftrightarrow \Delta Y.v = s.Y - \delta.K$ (eq.2)

$\Leftrightarrow \Delta Y = (s.Y - \delta.K)/v$

$\Leftrightarrow \Delta Y/Y = (s.Y - \delta.K)/(v.Y)$

$\Leftrightarrow \Delta Y/Y = s.Y/(v.Y) - \delta.K/(v.Y)$

$$\Leftrightarrow g^w = (s/v) - \delta$$

Harrod-Domar model: some implications of the reduced-form equation

$$g^w = (s/v) - \delta$$

- The growth rate of the economy will be greater, the greater the savings rate and the smaller the capital-output ratio and the depreciation rate (s, v and δ all being exogenous to the model).
- The savings rate, as a behavioural variable, is the most obvious policy tool on this model: in order to grow faster, economies should save more (a very non-Keynesian conclusion, by the way). The capital-output ratio and the depreciation rate are technical variables and somewhat less amenable to policy manipulation, although governments can and should try to promote technological progress (a smaller v).
- Poor countries are stuck in a "poverty trap": their income is low, therefore savings are low, the capital stock increases slowly, etc. Overcoming this trap requires increasing the savings rate significantly (hard to achieve in a poor economy) or access to foreign savings (through borrowing, foreign aid or foreign investment).

Harrod-Domar model: an unstable equilibrium

$$g^w = (s/v) - \delta \quad \rightarrow \quad \text{why } "g^w"?$$

The w in g^w stands for ***warranted*** *("garantida")*. It means this is the growth rate made possible by the given values of s, v and δ as long as all the hypotheses hold all the time – in particular, the hypotheses that productive capacity is fully used, that savings are fully invested and that the required technology which converts K into (1/k).Y always applies.

What happens if or whatever reason (business cycle, lack or excess of confidence, other exogenous shocks, etc) these hypotheses do not hold?

Harrod-Domar model: an unstable equilibrium

Two disequilibrium possibilities:

a. recessive disequilibrium:
if production decisions < feasible production with the existing stock of capital, then
$v^{ef} > v$ (reduction of the efficiency of the capital)
$r(Y^e) < r^w(Y)$ (actual growth rate < *warranted* growth rate)
Excess of capital goods → income grows more slowly than the productive capacity of the economy → disequilibrium and unemployment

b. inflationary disequilbrium:
if production decisions > feasible production with the existing stock of capital, then
$v^{ef} < v$ (increase in the efficiency of the capital)
$r(Y^e) > r^w(Y)$ (actual growth rate > *warranted* growth rate)
Shortage of capital → income grows faster than the productive capacity of the economy → disequilibrium and inflation

Harrod-Domar model: an unstable equilibrium

Key problem: there is no self-correcting mechanism in the event of a temporary deviation (knife-edge equilibrium).

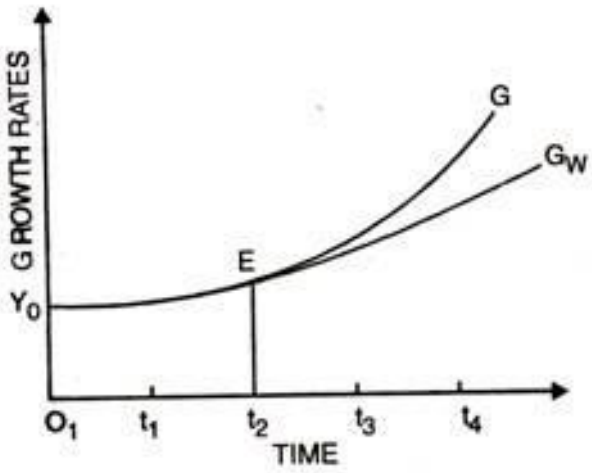


Fig. 1.

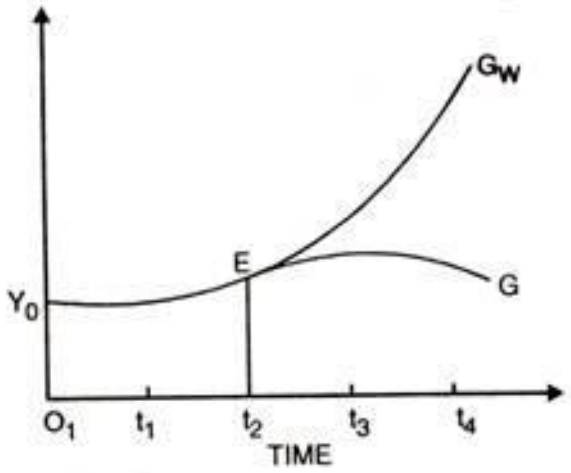
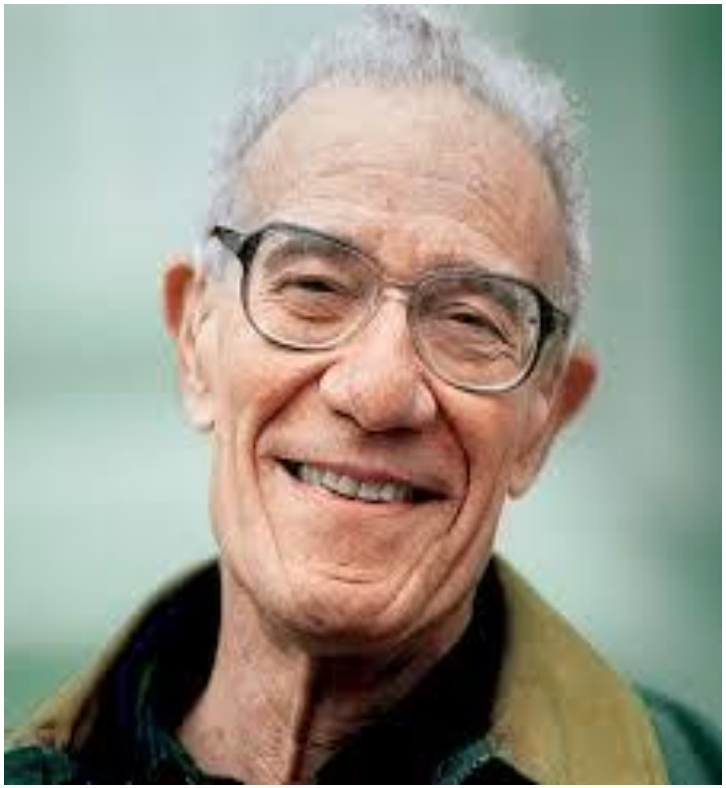


Fig. 2.

The critical interpretation by R. Solow:



"a certain discomfort that I felt with their work. (…) Harrod and Domar seemed to be answering a straightforward question: when is an economy capable of steady growth at a constant level? They arrived by noticeable routes, at a classical simple answer: the national saving rate (the fraction of income saved) has to be equal to the product of the capital output ratio and the rate of growth of the (effective) labor force. Then and only then could the economy keep its stock of plant and equipment in balance with its supply of labor, so that steady growth could go on without the appearance of labor shortage on one side or labor surplus and growing unemployment on the other side. **They were right about that general conclusion**."

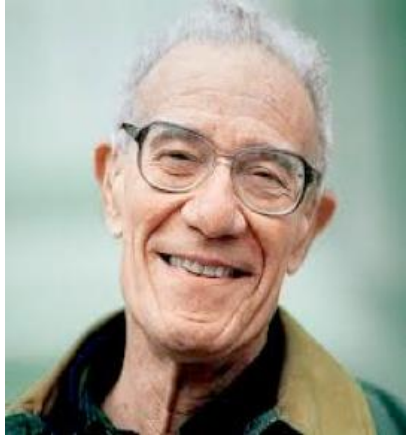
But…

Solow's criticisms to the hypotheses of H-D model

For there always to be constant full employment,
$r(L) = g^w = (s/v) - \delta$

However, the savings rate (*preferences*), the growth rate of labour supply (*demographic and sociological*) and the capital-output ratio (*technology*) in the H-D model are exogenous and, in fact, independent.

They may change but independently: they are not related to each other. **"In that case (…) the possibility of steady growth would be a miraculous stroke of luck"** (Solow)



Conclusion by R. Solow:

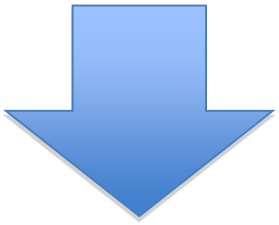
"***That was the spirit in which I began tinkering with the theory of economic growth, trying to improve on the Harrod-Domar model. I can not tell you why I thought first about replacing the constant capital-output (and labor-output) ratio by a richer and more realistic representation of the technology***"

- suggesting an alternative: the neoclassical production function (with technological flexibility and diminishing returns to capital);
- allowing for a varying capital-output ratio (productivity) which depends on population as well as the capital stock;
- and allowing for internal adjustment mechanisms which brings the economy back into equilibrium in the event of an exogenous shock.

…. and solutions by R. Solow

technological flexibility through the **substitutability** of production factors (not complementarity); therefore, the adjustment occurs within the productive process

v (capital-output ratio) is not a constant but, instead, becomes a variable (internal adjustment)



The Solow-Swan model